STUDY OF DISTURBANCE FRONT PROPAGATION IN MULTILAYERED MEDIUM

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We examine the propagation of a disturbance front in a multilayered medium separated by locally permeable zones. Relations are derived which make it possible to determine for different regimes the position of the moving boundary and the pressure at the outer impermeable boundary. It is shown that for certain values of the layer parameters the disturbance front may not reach the natural boundary. It is found that if the disturbance front propagates to the outer boundary, then at this boundary there is a limiting value of the pressure after which it will not change. Formulas are obtained for finding the limiting length of the disturbance front and the limiting value of the pressure at the outer boundary.

The study of disturbance front propagation in liquid and gas motion in a multilayered medium is of great importance in hydrogeology, and also in the exploitation of oil and gas deposits [1, 2]. The exact mathematical study of these questions involves tremendous difficulties. Even for the simplest problems, which are amenable to exact solution, the formulas obtained are complex in form and it is very difficult to carry out numerical calculations using them and draw any practical conclusions. Therefore, the development of special approximate methods [2-4] and the use of computers for the solution of liquid and gas flow problems in multilayered media are particularly important [5-7].

Assume a battery of wells, replaceable by a gallery, is put into operation with the constant flowrate q in a semi-infinite stratum of thickness h and permeability k. At the initial time the pressure at any point of the stratum is constant and equal to p_0. The stratum floor is considered impermeable. Above this stratum there extends another stratum, which is separated from the first by a relatively impervious layer and has the constant pressure p°. The relatively impermeable layer between the strata contains locally permeable zones. It is assumed initially that there is only a single locally permeable zone of width (a_2-a_1) with permeability k^* and thickness b^*.

For the exact mathematical solution of this problem we must integrate the differential equation

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\frac{1}{\varkappa}\frac{\partial p}{\partial t} \tag{1}$$

for the two-dimensional region with the boundary and initial conditions

$$\frac{\partial p}{\partial y}=0 \text{ for } \begin{cases} y=0, & 0\leqslant x\leqslant\infty \\ y=h, & 0\leqslant x\leqslant a_1 \\ y=h, & a_2\leqslant x\leqslant\infty \end{cases} \tag{2}$$

$$p=p^\circ \quad \text{for } y=h+b^*,\ a_1<x<a_2$$

$$\frac{k}{\mu}h\frac{\partial p}{\partial x}=q \text{ for } x=0,\quad p=p_0 \quad \text{for} \quad t=0$$

The piezoconductivity $\varkappa$ and the other coefficients are assumed to be constant.

The described problem is solved using approximate methods, the essence of which is that the entire region is broken down into perturbed and unperturbed zones. The pressure distribution law in the perturbed zone is specified, and in the unperturbed zone it is taken to be the original distribution law [2-4].

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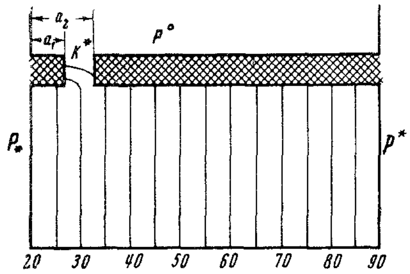

Fig. 1

We examine the first pressure redistribution phase, and the distribution law in the disturbed zone is taken in the form

$$p = p_0 - \frac{\mu l q}{2kh}\left(1 - \frac{x}{l}\right)^2 \tag{3}$$

This equation satisfies the conditions $\partial p / \partial x = 0$, $p = p_0$ for $x = l\,(t)$.

Here $l\,(t)$ is the disturbance front length measured from the gallery, and μ is the liquid viscosity.

The quantity $l\,(t)$ in (3) is found from the material balance equation.

Let dq_1 be the liquid mass withdrawn by the gallery during the time interval dt. Then

$$dq_1 = q\,dt \tag{4}$$

The amount of liquid obtained as a result of elastic storage is found from

$$q_2 = \beta h l\,(p - p_*^\circ) \tag{5}$$

Here β is the elastic capacity coefficient of the stratum and liquid.

On the basis of (3) the weighted mean pressure p_*° is found in the form

$$p_*^\circ = p_0 - \frac{\mu q l}{6kh} \tag{6}$$

From (5) and (6) we obtain

$$dq_2 = \frac{q}{3\varkappa}\,dl \tag{7}$$

As is known, the question of account for the amount of fluid which leaks from one stratum into the other is important in the solution of flow problems in multilayered media [2].

Let the amount of fluid leaking through the locally permeable zone of the relative impermeable layer during the time interval dt be dq_3. Then

$$\frac{dq_3}{dt} = Q_v = \frac{\alpha q}{2l}\left[l^2 (a_2 - a_1) - l\,(a_2^2 - a_1^2) + {}^1\!/_3\,(a_2^3 - a_1^3)\right], \qquad \alpha = \frac{k^*}{khb^*} \tag{8}$$

In obtaining (8) we have assumed that $p_0 = p^\circ = 0$. We see from (8) that the amount of crossflow fluid depends not only on the parameters of the active stratum, but also on the parameters of the relatively impermeable layer. Since

$$dq_1 = dq_2 + dq_3 \tag{9}$$

we obtain from (4), (7), and (8)

$$\varkappa dt = \frac{l\,dl}{3\left\{1 - {}^1\!/_2\,\alpha l^{-1}\left[l^2 (a_2 - a_1) - l\,(a_2^2 - a_1^2) + {}^1\!/_3\,(a_2^3 - a_1^3)\right]\right\}} \tag{10}$$

Thus we have obtained the differential equation for determining the unknown $l\,(t)$ when the relatively impermeable layer has only a single locally permeable zone of width $(a_2 - a_1)$. Integration of (10) in the limits from 0 to t and from 0 to l, respectively, leads to the expression

$$\frac{\varkappa t}{2} = -\frac{l}{3\alpha (a_2 - a_1)} + \frac{B}{18\alpha^2 (a_2 - a_1)} \ln \frac{C + 3\alpha l^2 (a_2 - a_1) - Bl}{C} + \frac{B^2 - 6\alpha^2 (a_2^3 - a_1^3)(a_2 - a_1)}{18\alpha^2 (a_2 - a_1)^2 A} \ln \frac{l\,(B + A) - 2C}{l\,(B - A) - 2C}$$

$$A = \left\{\left[6 + 3\alpha (a_2^2 - a_1^2)\right]^2 - 12\alpha^2 (a_2^3 - a_1^3)(a_2 - a_1)\right\}^{1/2}$$

$$B = 6 + 3\alpha (a_2^2 - a_1^2), \qquad C = \alpha (a_2^3 - a_1^3) \tag{11}$$

This equation makes it possible to determine the position of the moving boundary for any time. If we consider the distribution law (3) as well, we can find the pressure at any section of the stratum being exploited at any time.

We note that limiting regimes have been found in the study of fluid motion in multilayered media [2]. It has been established that for certain values of the parameters of the relatively impermeable layer and the exploited stratum, the disturbance front may not reach the natural boundary of the stratum. In other words, the limiting position of the disturbance front depends on the permeability and thickness of the stratum and of the less permeable layer. The existence of a disturbance front propagation limit was first shown by Polubarinova-Kochina in [8]. Further, if the parameters of the stratum and less permeable layer are such that the disturbance can reach the impermeable outer boundary of the stratum, there will be a limiting value of the pressure at the outer boundary, after which the pressure will not change. The idea of the existence of a limiting value of the pressure impermeable boundary was first proposed by Hussein-zade [2].

Approximate methods and computers have been used to determine, respectively, the maximal disturbance front travel and the limiting pressure at the outer impermeable boundary for various cases [9].

The limiting value of the disturbance front propagation for the case in question here is found from (10)

$$l_* = \frac{6 + 3\alpha (a_2^2 - a_1^2) + \{[6 + 3\alpha (a_2^2 - a_1^2)]^2 - 12\alpha^2 (a_2^3 - a_1^3)\}^{1/2}}{6\alpha (a_2 - a_1)} \tag{12}$$

From the formulas presented above, we can find the solutions for the case in which there is a fluid crossflow along the entire length of the relatively impermeable layer. To do this we substitute $a_1 = 0$ and $a_2 = l$ into the formulas obtained above. Then

$$l_* = \sqrt{6}\,\alpha^{-1/2}, \qquad l^2 = \frac{6}{\alpha}\,[1 - \exp(-2\varkappa\alpha t)]$$

These results coincide with the formula of [2]. Thus, we have obtained formulas which make it possible to determine the pressure at any section of the stratum at an arbitrary time for the case in which the disturbance has not yet reached the outer boundary of the stratum.

If the condition is specified at the boundary

$$p = p_* \quad \text{for} \quad x = 0$$

then the solution of the described problem is obtained in the form

$$\varkappa t = -\frac{l}{3\alpha (a_2 - a_1)} + \frac{B}{18\alpha^2 (a_2 - a_1)} \ln \frac{3l^2 (a_2 - a_1)\alpha + C - Bl}{C} + \frac{B^2 - 6\alpha^2 (a_2^3 - a_1^3)(a_2 - a_1)}{18\alpha^2 (a_2 - a_1)^2 A} \ln \frac{l(B + A) - 2C}{l(B - A) - 2C} \tag{13}$$

Formula (13) is obtained for

$$p = p_0 - (p_0 - p_*)(1 - x/l)^2 \tag{14}$$

Equation (14) satisfies the conditions

$$p = p_0 \quad \text{for} \quad x = l(t), \qquad p = p_* \quad \text{for} \quad x = 0$$

The limiting value of the disturbance front length for this case is also found using (12).

It follows from these formulas that, regardless of the operating regime, the maximal travel of the boundary of the disturbed and undisturbed zones has the same expression. However, this limiting distance is reached at different times for each of the cases in question. Actually, we see from (11) and (13) that the maximal value of the disturbance front length, when operating in the constant pressure regime, is reached twice as fast as in the constant flowrate regime. This circumstance is also of practical interest in that after a definite period of time the gallery will be essentially supplied only by the upper stratum, located above the relatively impermeable layer.

We note that there are no terms characterizing the pressure change through the thickness of the stratum in the pressure distribution relations used or in the approximate formulas used in [2] to solve flow problems in a multilayered medium. This is explained by the fact that under natural conditions the permeability of the relatively impermeable layer is much less than that of the stratum being exploited [2]. Therefore, for very large values of the ratio k/k* we can neglect the pressure change in the vertical direction. Figure 1 presents the results of computer experiments conducted for

$$p^* = 90\,\text{atm}, \; p^\circ = 40\;\text{atm}, \; p_* = 20\,\text{atm}, \; a_1 = 3\;\text{m}$$
$$a_2 = 6\,\text{m}, \; k^* : k = 0.2 \;\; L = 33\,\text{m}, \; h = 15\,\text{m}, \; b^* = 3\,\text{m}$$

where p* is the pressure in the injection gallery, and p_* is the pressure in the active gallery.

The results shown in Fig. 1 were obtained for an analogous case with steady flow. We see from Fig. 1 that even for k=5k* the equipotential lines will be nearly straight. The accuracy of the approximate methods used has been evaluated several times [2, 9, 10].

If the disturbance reaches the outer boundary of the stratum, the second redistribution phase starts. Then

$$l(t) = L = \text{const}, \qquad p_0 = p_\infty(t) = \text{var}$$

Here L is the distance from the gallery to the outer edge of the stratum, and $p_\infty(t)$ is the pressure at the outer impermeable boundary of the stratum.

Proceeding as above, we can find that

$$dq_2 = -\beta h L^2/_3\, dp_\infty \tag{15}$$

$$dq_3 = {}^1/_2 \alpha q L^{-1} [L^2 (a_2 - a_1) - L (a_2^2 - a_1^2) + {}^1/_3 (a_2^3 - a_1^3)]\, dt \tag{16}$$

Using (4), (15), (16) and considering (9), we obtain

$$qdt \{1 - {}^1/_2 \alpha L^{-1} [L^2 (a_2 - a_1) - L (a_2^2 - a_1^2) + {}^1/_3 (a_2^3 - a_1^3)]\} = -\beta h L dp_\infty \tag{17}$$

Integration of (17) in the limits from p_0 to p_∞ and from 0 to t, respectively, leads to the solution

$$p_\infty(t) = p_0 - \frac{qt}{\beta h L}\left\{1 - \frac{\alpha}{2L}\left[L^2 (a_2 - a_1) - L (a_2^2 - a_1^2) + \frac{a_2^3 - a_1^3}{3}\right]\right\} \tag{18}$$

If the fluid leaks along the entire length of the relatively impermeable layer, we obtain as a particular case from (18)

$$p_\infty(t) = p_0 - \frac{qt}{\beta h L}\left(1 - \frac{\alpha L^2}{6}\right) \tag{19}$$

which coincides with the formula of [2]. If the active stratum does not have any connections with the overlying stratum ($\alpha = 0$), then (19) takes the form

$$p_\infty(t) = p_0 - \frac{qt}{\beta h L} \tag{20}$$

For the case in which the pressure in the gallery is given, the equations for finding d and dq_3 take the form

$$dq_1 = \frac{kh}{\mu}\frac{2(p_\infty - p_*)}{L} dt \tag{21}$$

$$dq_3 = \frac{k^*}{b^*\mu}\left\{p_0 (a_2 - a_1) + p_\infty\left[\frac{a_2^3 - a_1^3}{3L^2} - \frac{a_2^2 - a_1^2}{L}\right] - p_*\left[(a_2 - a_1) - \frac{a_2^2 - a_1^2}{L} + \frac{a_2^3 - a_1^3}{3L^2}\right]\right\} dt \tag{22}$$

Using (15), (21), and (22), we obtain

$$\varkappa dt = -\frac{2L dp_\infty}{p_\infty B(\alpha) - 3\alpha p_0 (a_2 - a_1) + p_* [3\alpha (a_2 - a_1) - B(\alpha)]} \tag{23}$$

$$B(\alpha) = \frac{3\alpha (a_2^2 - a_1^2)}{L} + \frac{6}{L} - \frac{3\alpha (a_2^3 - a_1^3)}{3L^2}$$

The limiting value of the pressure at the outer impermeable boundary is found in the form

$$p_\infty^\circ = p_* B^{-1}(\alpha) [B(\alpha) - 3\alpha (a_2 - a_1)] + 3B(\alpha) p_\infty \alpha (a_2 - a_1) \tag{24}$$

For the case in which the fluid leaks along the entire length of the relatively impermeable layer ($a_2 = L$, $a_1 = 0$, $p_0 = 0$), (24) takes the form [2]

$$p_\infty^\circ = \frac{p_*}{3 + \alpha L^2}\left(3 - \frac{\alpha L^2}{2}\right) \tag{25}$$

Integration of (23) from 0 to t and from p_0 to p_∞, respectively, yields

$$p_\infty(t) = p_*\left[1 - \frac{3\alpha (a_2 - a_1)}{B(\alpha)}\right]\left(1 - \exp\frac{-B(\alpha)\varkappa t}{2L}\right) \tag{26}$$

If the active stratum is connected with the overlying stratum through a large number of locally permeable zones, the solution will be obtained in the form

$$p_\infty(t) = p_*\left\{1 - \frac{3\alpha[(a_2 - a_1) + \cdots + (a_{2n} - a_{2n-1})]}{B(\alpha)}\right\}\left(1 - \exp\frac{-B(\alpha)\varkappa t}{2L}\right) \quad (27)$$

Here n takes positive integral values. For the case of fluid crossflow along the entire length of the relatively impermeable layer ($a_1 = 0$, $a_2 = L$, ..., $a_{2n} = a_{2n-1}$), we have the solution obtained previously [2]

$$p_\infty(t) = \frac{p_*}{3 + \alpha L^2}\left(3 - \frac{\alpha L^2}{2}\right)\left[1 - \exp\frac{-(\alpha L^2 + 3)\varkappa t}{L^2}\right] \quad (28)$$

However, if there is no connection between the active and overlying strata, (28) takes the form

$$p_\infty = p_*\left(1 - \exp\frac{-3\varkappa t}{L^2}\right)$$

Thus, we have obtained simple formulas for determining the moving boundary location at any time and the pressure variation at the outer impermeable boundary.

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